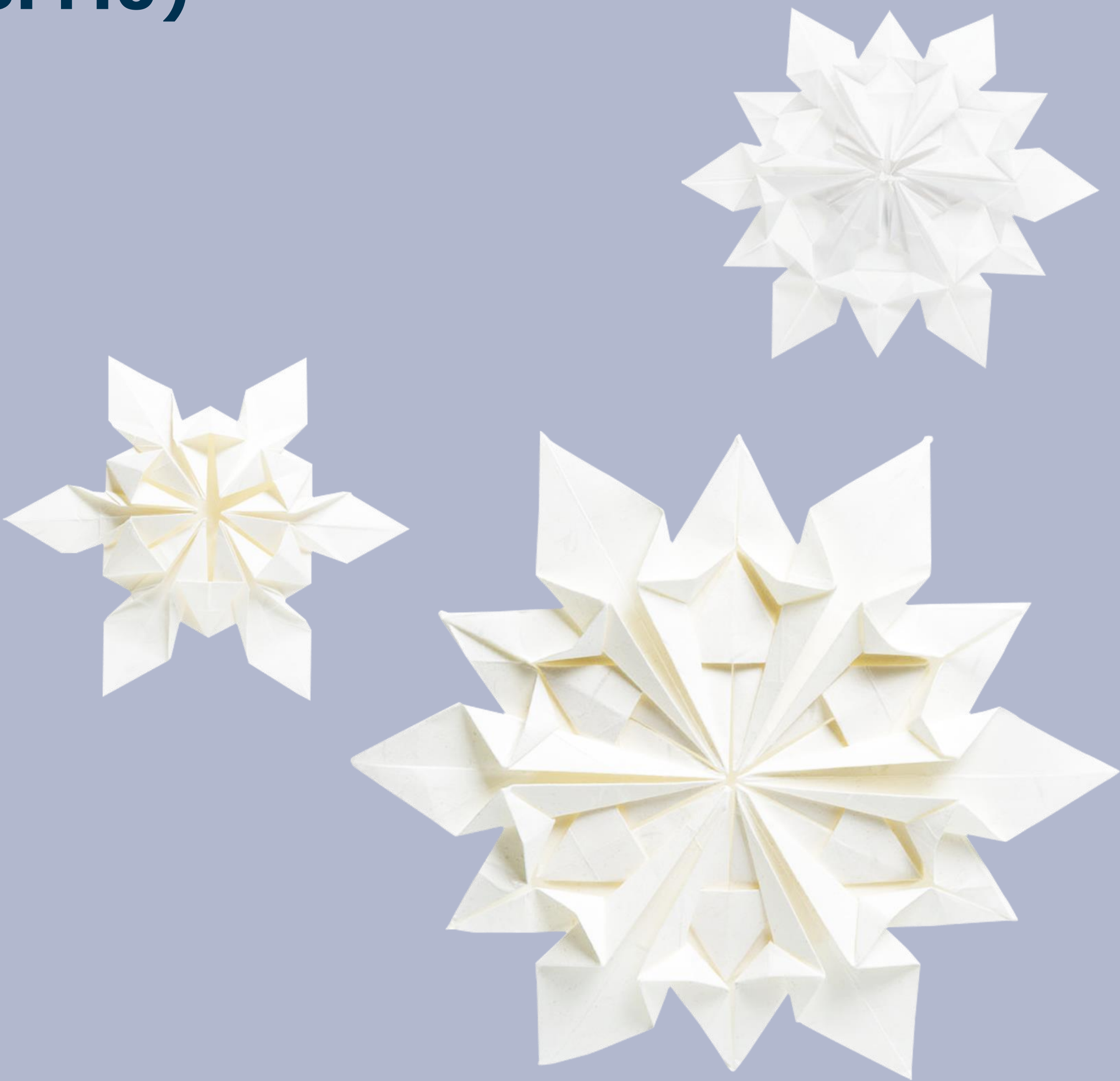
Pearson Edexcel
Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)



Sample Assessment Materials Model Answers – Further Mechanics 1&2

First teaching from September 2017
First certification from June 2018

Sample Assessment Materials Model Answers – Further Mechanics 1&2

Contents

Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8FMO) specification for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS Further Mathematics qualification, specifically at further mechanics 1 and 2 questions, and is intended to offer model solutions with different methods explored.

Content of Further Mechanics 1&2

Content	**AS level content**
	Further Mechanics 1
Momentum and impulse	Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two spheres colliding directly.
Work, energy and power	Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.
Elastic collisions in one dimension	Direct impact of elastic spheres. Newton's law of restitution. Loss of kinetic energy due to impact. Successive direct impacts of spheres and/or a sphere with a smooth plane surface.
	Further Mechanics 2
Motion in a circle	Angular speed $v = r\omega$ Radial acceleration in circular motion. The forms $r\omega^2$ and $\frac{v^2}{r}$ are required. Uniform motion of a particle moving in a horizontal circle.
Centres of mass of plane figures	Moment of a force. Centre of mass of a discrete mass distribution in one and two dimensions. Centre of mass of uniform plane figures, and simple cases of composite plane figures. Centre of mass of frameworks. Equilibrium of a plane lamina or framework under the action of coplanar forces.
Further kinematics	Kinematics of a particle moving in a straight line when the acceleration is a function of the time (t) or velocity (v).

AS Further Mechanics 1

Question 1

A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5.

(6)

$v^2 = u^2 + 2as$		
$v^2 = 0^2 + 2g \times 2.4$		M1
$v^2 = 4.8g$		
$v = \sqrt{4.8g}$	(= 6.86)	A1
$v^2 = u^2 + 2as$		
$0^2 = u^2 - 2g \times 0.6$		M1
$u^2 = 1.2g$		
$u = \sqrt{1.2g}$	(= 3.43)	A1

Newton's Law of Restitution (NLR):

e = speed of separation ÷ speed of approach = $\dfrac{u}{v}$

$e = \dfrac{\sqrt{1.2g}}{\sqrt{4.8g}}$	M1
$e = 0.5$	A1

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

speed of separation = e × speed of approach

$u = 0.5 \times \sqrt{1.2g}$	(= 1.71)	M1
$v^2 = u^2 + 2as$		
$0^2 = 0.5^2(1.2g) - 2gh$		M1
$2gh = 0.3g$		
$h = 0.15$ m		A1

(c) By considering your answer to part (b), describe the subsequent motion of the ball. **(1)**

The ball continues to bounce with the height of each bounce being a quarter of the previous one.
B1

Question 2

A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

Energy loss = KE loss – PE gain	M1
$= \frac{1}{2} mv^2 - mgh$	
$= \frac{1}{2}\times 0.5\times 25^2 - 0.5g\times 20$	A1
$= 58.25$	
$= 58.3$ J	A1

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

Using the work-energy principle work done = force × distance	
$58.25 = R \times 20$	M1
$R = 2.9125$	
$R = 2.91$	A1

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Make the resistance variable (dependent on speed)	B1

Question 3

[In this question use $g = 10 \text{ m s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of 4 m s^{-1}. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

Force = Resistance = 30 (since no acceleration)	B1
Power = Force × Speed	
$P = Fv$	
$= 30 \times 4$	M1
$= 120$ W	A1

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{15}$

Because of the hill, the jogger reduces her speed to 3 m s^{-1} and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at 3 m s^{-1}.

(5)

Resolving parallel to the slope	
$F - 60g \sin \alpha - 30 = 0$	M1
	A1
$F = 60 \times 10 \times \frac{1}{15} + 30$	
$F = 70$	A1
$P = Fv$	
$= 70 \times 3$	M1
$= 210$ W	A1

Question 4

A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table.

A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Using conservation of momentum (CLM)

$(3m)u - m(2u) = (3m)v + mw$ M1

$3mu - 2mu = 3mv + mw$ A1

$u = 3v + w$ ---[1]

Using Newton's Law of Restitution (NLR) speed of approach × e = speed of separation

$(u + 2u) \times e = w - v$ M1

$3ue = -v + w$ ---[2] A1

To eliminate w, subtract: [1] – [2]

$u - 3ue = 3v + v$

$4v = u(1 - 3e)$ M1

$v = \frac{u}{4}(1 - 3e)$ A1

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

If the direction of motion is changed by the collision, $v < 0$

$\frac{u}{4}(1 - 3e) < 0$ M1

$1 - 3e < 0$

$e > \frac{1}{3}$

but $e \leq 1$ always

$\frac{1}{3} < e \leq 1$ A1

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Either:	**Or:**	
Substitute $v = \frac{u}{4}(1 - 3e)$	Substitute $v = \frac{u}{4}(1 - 3e)$	
into $u = 3v + w$	into $3ue = -v + w$	
$u = \frac{3u}{4}(1 - 3e) + w$	$3ue = -\frac{u}{4}(1 - 3e) + w$	
$u = \frac{3}{4}u - \frac{9}{4}ue + w$	$3ue = -\frac{1}{4}u + \frac{3}{4}ue + w$	
$w = u - \frac{3}{4}u + \frac{9}{4}ue$	$w = \frac{1}{4}u + 3ue - \frac{3}{4}ue$	M1
$w = \frac{u}{4}(1 + 9e)$	$w = \frac{u}{4}(1 + 9e)$	A1

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Substitute $e = \frac{5}{9}$ into the equations for v and w:

$v = \frac{u}{4}(1 - 3(\frac{5}{9}))$	$w = \frac{u}{4}(1 + 9(\frac{5}{9}))$	M1
$v = \frac{u}{4}(-\frac{2}{3})$	$w = \frac{u}{4}(6)$	
$v = -\frac{u}{6}$	$w = \frac{3u}{2}$	A1

Let x = speed of Q after collision with wall.

Then using NLR, $x = f \times w$ M1

There will be a further collision if $x > -v$ M1

i.e. if $f \times w > -v$

$f \times \frac{3u}{2} > -(-\frac{u}{6})$ A1

$\frac{3}{2}f > \frac{1}{6}$ (since $u > 0$)

$f > \frac{1}{9}$

but $f \leq 1$ always

$\frac{1}{9} < f \leq 1$ A1

AS Further Mechanics 2

Question 5

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geq 0.$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

$v = 3t^2 - 16t + 20$

$\dfrac{dv}{dt} = 6t - 16$ M1

$a = \dfrac{dv}{dt}$

$a = 6t - 16$ A1

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

$s = \int v \, dt$

$s = \int 3t^2 - 16t + 20 \, dt$

$s = t^3 - 8t^2 + 20t + c$ M1 A1

When $t = 0$, $s = 0$, so $c = 0$

$s = t^3 - 8t^2 + 20t$

When $t = 2$,

$s = 8 - 32 + 40$

$s = 16$ A1

(c) Show that P never returns to O, explaining your reasoning. **(3)**

If P returns to O, then $s = 0$.
$t^3 - 8t^2 + 20t = 0$

Divide through by t since $t \neq 0$
$t^2 - 8t + 20 = 0$ M1

Complete the square:
$(t - 4)^2 - 16 + 20 = 0$
$(t - 4)^2 + 4 = 0$

This is not possible, $(t - 4)^2 + 4 \geq 4$ for all values of t. M1

So $s = 0$ has no non-zero solutions, so s is never zero again,
so never returns to O. A1

Question 6

A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$.

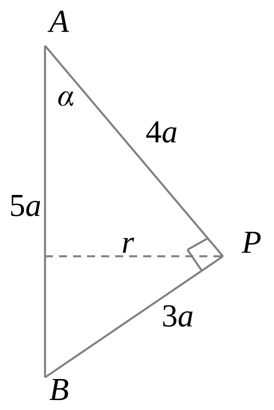
A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

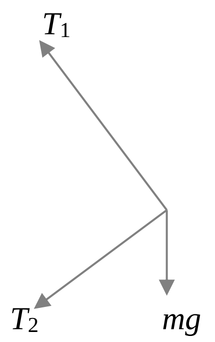
(a) Show that

(i) the tension in AP is $\dfrac{4m}{25}(9a\omega^2 + 5g)$,

(ii) the tension in BP is $\dfrac{3m}{25}(16a\omega^2 - 5g)$.

(10)





There is a right angle at P, so $\sin\alpha = \dfrac{3}{5}$	B1
and $\cos\alpha = \dfrac{4}{5}$	
Radius of horizontal circle, $r = 4a\sin\alpha$	B1
$r = 4a \times \dfrac{3}{5} = \dfrac{12}{5}a$	
Resolving vertically:	
$T_1\cos\alpha = T_2\sin\alpha + mg$ ---[1]	M1
	A1
Radial acceleration:	
$a = r\omega^2$	
Resolving horizontally using $F = ma$:	
$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$ ---[2]	M1
	A2

(i)
Rearrange [1]:

$$T_2 = \frac{T_1 \cos\alpha - mg}{\sin\alpha}$$

sub into [2]:

$$T_1 \sin\alpha + \frac{(T_1 \cos\alpha - mg)\cos\alpha}{\sin\alpha} = mr\omega^2$$

$$\frac{3}{5}T_1 + \frac{\left(\frac{4}{5}T_1 - mg\right)\frac{4}{5}}{\frac{3}{5}} = m\left(\frac{12}{5}a\right)\omega^2$$

$$\frac{9}{5}T_1 + \frac{16}{5}T_1 - 4mg = \frac{36}{5}ma\omega^2$$

$$5T_1 = \frac{36}{5}ma\omega^2 + 4mg$$

$$T_1 = \frac{36}{25}ma\omega^2 + \frac{20}{25}mg$$ M1

$$T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$$ A1

(ii)
Sub into rearranged [1]:

$$T_2 = \frac{T_1 \cos\alpha - mg}{\sin\alpha}$$

$$T_2 = \frac{\frac{4}{25}m(9\alpha\omega^2 + 5g)\frac{4}{5} - mg}{\frac{3}{5}} = \frac{16}{75}m(9a\omega^2 + 5g) - \frac{5}{3}mg$$

$$T_2 = \frac{144}{75}ma\omega^2 + \frac{80}{75}mg - \frac{125}{75}mg = \frac{48}{25}ma\omega^2 - \frac{15}{25}mg$$

$$T = \frac{3m}{25}(16a\omega^2 - 5g)$$ A1

The string will break if the tension in it reaches a magnitude of $4mg$.
The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

$T_1 < 4mg$

$\frac{4m}{25}(9a\omega^2 + 5g) < 4mg$ ---[1] M1

$T_2 > 0$

$\frac{3m}{25}(16a\omega^2 - 5g) > 0$ ---[2] M1

[1]:

$9a\omega^2 + 5g < 25g$

$\omega^2 < \frac{25g - 5g}{9a}$

$\omega < \sqrt{\frac{20g}{9a}}$ A1

$\omega < \frac{2}{3}\sqrt{\frac{5g}{a}}$

[2]:

$16a\omega^2 - 5g > 0$

$\omega^2 > \frac{5g}{16a}$

$\omega > \sqrt{\frac{5g}{16a}}$

$\omega > \frac{1}{4}\sqrt{\frac{5g}{a}}$

$S = \frac{2\pi}{\omega}$

$S > \frac{2\pi}{\frac{2}{3}\sqrt{\frac{5g}{a}}}$ M1

$S > 3\pi\sqrt{\frac{a}{5g}}$

$S = \dfrac{2\pi}{\omega}$

$S < \dfrac{2\pi}{\frac{1}{4}\sqrt{\frac{5g}{a}}}$

$S < 8\pi\sqrt{\dfrac{a}{5g}}$

$3\pi\sqrt{\dfrac{a}{5g}} < S < 8\pi\sqrt{\dfrac{a}{5g}}$ A1

(c) State how in your calculations you have used the assumption that the string is light. **(1)**

The string being light implies that the tension is constant in both portions of the string. B1

Question 7

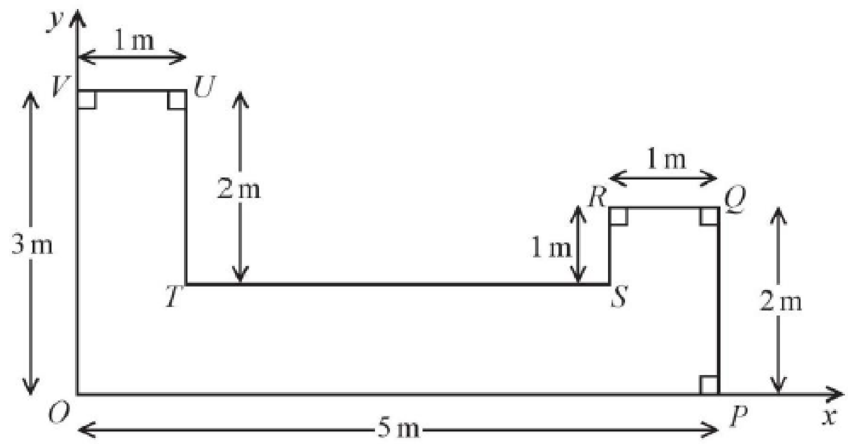


Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.

$OP = 5$ m, $PQ = 2$ m, $QR = 1$ m, $RS = 1$ m, $TU = 2$ m, $UV = 1$ m, $VO = 3$ m.

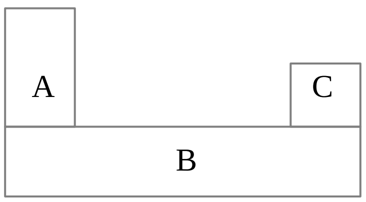
Figure 1 also shows a coordinate system with O as origin and the x-axis and y-axis along OP and OV respectively. The unit of length on both axes is the metre. The centre of mass of the template has coordinates ($\bar{x}$, $\bar{y}$).

(a) (i) Show that $\bar{y} = 1$,

(ii) Find the value of $\bar{x}$.

(7)

Split the template into 3 rectangles:



Area of rectangles $\rightarrow$ Relative mass.

	A	B	C	Total
Relative Mass	2	5	1	8
y	2	0.5	1.5	$\bar{y}$
x	0.5	2.5	4.5	$\bar{x}$

R.M's B1
y's B1
x's B1

(i)

$(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$ M1

$\bar{y} = 1$ A1

(ii)

$(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$ M1

$\bar{x} = 2.25$ A1

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius r metres, are removed from the template which is shown in Figure 1, to form a new template L. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, a) where a is a constant.

(b) Find the value of r.

(4)

	Template	Circle 1	Circle 2	Total
Relative Mass	8	πr^2	πr^2	$8 - 2\pi r^2$
x	2.25	0.5	0.5	2.5

$(8\times2.25) - (\pi r^2\times0.5) - (\pi r^2\times0.5) = (8 - 2\pi r^2)\times2.5$ M1 A1

$18 - \pi r^2 = 20 - 5\pi r^2$
$4\pi r^2 = 2$

$r = \dfrac{1}{\sqrt{2\pi}}$ M1

$r = 0.399$ A1

(c) (i) Explain how symmetry can be used to find the value of a.

(ii) Find the value of a.

(2)

(i)
Since the $\bar{y}$ centre of mass for the original plate is also 1, the holes must be symmetrically placed about the line $y = 1$. B1

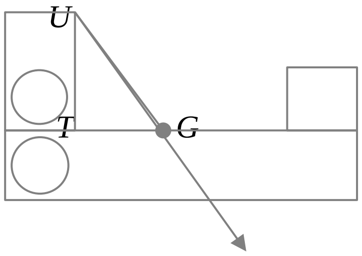
(ii)
$a = 1 + 0.5 = 1.5$ B1

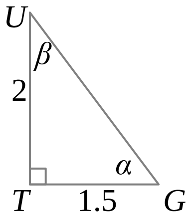
The template L is now freely suspended from the point U and hangs in equilibrium.

(d) Find the size of the angle between the line TU and the horizontal.

(3)

G = centre of mass





Angle between line TU and the vertical = β, so
Angle between line TU and the horizontal = α

$\tan \alpha = \frac{2}{1.5} = \frac{4}{3}$ M1A1

$\alpha = 53.1^{\circ}$ A1

For more information on Edexcel and BTEC qualifications please visit our websites:
www.edexcel.com and www.btec.co.uk

Edexcel is a registered trademark of Pearson Education Limited

Pearson Education Limited. Registered in England and Wales No. 872828
Registered Office: 80 Strand, London WC2R 0RL.
VAT Reg No GB 278 537121